# Assignment #7 – Machine Learning – Professor Haugh

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## Question 1

1. We have the following distribution

It can be re-written as,

Characterizing this function as an exponential family of the form

We can identify,

1. Substituting in the expression for the conjugate prior,

In order to transform this expression to the form,

We have to create the following equivalences,

From the first equation,

From the second relationship,

From here, we can state the following equations and solutions,

## Question 2

1. Let x follow a Bernoulli distribution, with parameter.
2. If the conjugate prior for is a beta distribution, it must have the form.
3. Computing the posterior distribution, considering i.i.d realizations of the r.v.

We recognize from here that the posterior is

## Question 3

1. We start with the expression

Using the definition of, and the variance function of correlated values,

Using the definition of the autocorrelation function, over the,

We know that, and, therefore,

From the definition of,

1. If we assume that follows an AR(1) process, , and .

From the definition of, and using this particular case for

From the definition of known series,

As n grows, becomes smaller and smaller.

We will assume n is large, yielding the convenient approximation,

1. We will start on the left hand side of the relationship we want to prove, and using the identity provided in the homework,

Using (2),

Since the are i.i.d. ,

With this result, we can re-arrange the terms

If we consider as data points, we can remove the expectation, yielding an estimator for,

This is an unbiased estimator only if is an unbiased estimator for. This condition must be true because if we take expectation of the right hand side, it will yield zero only if.

1. From the definition of each of the terms, and the result from part (b),

We first define an estimator using one of the chains, indexed by,

Where is the sample mean for chain.

Then,

Where

Finally, the unbiased estimator, using the data from the chains is,

Where,